THE MASSES OF GAUGE FIELDS IN HIGHER SPIN FIELD THEORY ON ADS(4)

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Abstract

Higher spin field theory on AdS(4) is defined by lifting the minimal conformal sigma model in three dimensional flat space. This allows to calculate the masses from the anomalous dimensions of the currents in the sigma model. The Goldstone boson field can be identified.

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1 Introduction

We consider the higher spin field theory constructed on AdS(4)space by lifting of the O(N) vector minimal sigma model from R(3)[1]. It results a nonlocal interacting renormalized QFT denoted HS(4) and defined by its n-point functions which can each be computed by a 1/N expansion. Such AdS field theory has the unconventional property that mass renormalizations of the fundamental fields and composite fields can be calculated perturbatively. This is made possible by the fact that the corresponding flat conformal field theory permits to calculate the anomalous part η of the conformal dimension Δ by 1/N expansion and that the mass m^2 of the AdS field and Δ of the dual conformal field are related, e.g. for symmetric tensor fields of rank l by

$$m_l^2 = \Delta(\Delta - d) - (l-2)(d+l-2) \tag{1}$$

Obviously for conserved currents $J^{(l)}(x)$ with exact dimension

$$\Delta(l) = d + l - 2 \tag{2}$$

we obtain a vanishing mass corresponding to the gauge fields $h^{(l)}$ on AdS space.

Lifting of the flat conformal field theory is done with the bulk-to-boundary propagator (say, in the scalar case, see [2] for the general case)

$$K_\Delta(z, \vec{x}) = \left(\frac{z_0}{z_0^2 + (\vec{z} - \vec{x})^2} \right)^\Delta \quad (z \in AdS(4), \vec{x} \in R(3)) \tag{3}$$

which satisfies the free field equation

$$(D_z - m^2) K_\Delta(z, \vec{x}) = 0 \tag{4}$$

with m^2 as in (1) and D_z a covariant second order differential operator. It is this equation which transforms dimension into mass.

If the symmetric tracesless tensor current $J^{(l)}(\vec{x})$ assumes an anomalous dimension $\eta(l)$ (which it does for $l \geq 4$)

$$\Delta(l) = d + l - 2 + \eta(l) \tag{5}$$

which can be 1/N expanded as

$$\eta(l) = \sum_{r=1}^{\infty} \frac{\eta_r(l)}{N^r} \tag{6}$$

we obtain masses for $h^{(l)}$ from (1) and (5)

$$m_l^2 = \eta(l)[d + 2(l-2)] + \eta(l)^2 \tag{7}$$

M. Porrati [3] has shown that a Higgs phenomenon is possible for a graviton to assume a mass. A massless symmetric tensor representation of the conformal group $[\Delta, l]$ with

$$\Delta = d - 2 + l|_{d=3} = l + 1 \tag{8}$$

which is massless, appears from an irreducible massive representation in the limit

$$\lim_{\Delta \to l+1} [\Delta, l] = [l+1, l] \oplus [l+2, l-1] \tag{9}$$

The second representation $[l{+}2, l{-}1]$ is identified with the Goldstone field. L. Girardello et al.[4] propose to create the Goldstone field from tensoring the conserved current $[l{-}1, l{-}2]$ corresponding to $J^{(l-2)}$ or $h^{(l-2)}$ with the scalar field $\alpha(\vec{x})$ on R(3) resp. $\sigma(z)$ on AdS(4) of dimension two (all in the free field limit)

$$[l-1, l-2] \otimes [2, 0] = \bigoplus_{s=0}^{\infty} \bigoplus_{n=0}^{\infty} [l+s+n+1, l+s-2] \tag{10}$$

which contains the Goldstone field representation for $s = 1$, $n = 0$. In the minimal sigma model the operator product of the current $J^{(l)}$ with the scalar field α contains by expansion the currents $J^{(l,t)}$ with dimension $d-2+l+2t$, $t \in \mathbf{N}$, (in the free field limit). Thus the Goldstone field of $J^{(l)}$ is $J^{(l-1,1)}$. A Higgs mechanism for producing the masses of the gauge fields is possible therefore except for the graviton since in (10) $l = 2$ is excluded (the representation $[1, 0]$ does not occur but is eliminated in favour of the dual representation $[2, 0]$ by the boundary condition of AdS(4)).

The gauge fields contain a traceless symmetric part $h^{(l)}$ and companion fields that are dynamically irrelevant. In HS(4) there exists a bilocal biscalar field $B(z_1, z_2)$ [1] which by operator product expansion decomposes into all $h^{(l)}$, $l \in 2\mathbf{N}$, and the scalar field $\sigma(z)$ at leading order in N, and further operators at order 1/N and higher. The starting point for a calculation of the masses m_l^2 is therefore the AdS four-point function

$$< B(z_1, z_3), B(z_2, z_4) >_{AdS} \tag{11}$$

2 The perturbative corrections to the bilocal fields

Instead of the AdS Green function (11) we study the flat CFT Green function

$$< b(\vec{x}_1, \vec{x}_3) b(\vec{x}_2, \vec{x}_4) >_{CFT} \tag{12}$$

where b is defined from the Lorentz scalar O(N) vector fields $\vec{\varphi}(\vec{x})$ by the normal product [1]

$$b(\vec{x}_1, \vec{x}_3) = N^{-1/2} \vec{\varphi}(\vec{x}_1) \vec{\varphi}(\vec{x}_3) \tag{13}$$

This O(N) vector field is normalized to

$$< \varphi_i(\vec{x}_1) \varphi_j(\vec{x}_2) >= \delta_{ij} (\vec{x}_{ij}^{\,2})^{-\delta} \qquad i, j \in 1, 2 \ldots N \tag{14}$$

where δ is the conformal dimension of φ (we set $d = 3$ at the end)

$$\delta = \mu - 1 + \eta(\varphi), \qquad \mu = \frac{d}{2} \tag{15}$$

$$\eta(\varphi) = \sum_{r=1}^{\infty} \frac{\eta_r(\varphi)}{N^r} \tag{16}$$

The first three terms in (16) are known [5].

Due to the contraction of O(N) vector indices the $O(1)$ contribution to the Green function is from the graphs A_1, A_3 with the result

$$(\vec{x}_{12}^{\,2} \vec{x}_{34}^{\,2})^{-\delta} + (\vec{x}_{14}^{\,2} \vec{x}_{23}^{\,2})^{-\delta} \tag{17}$$

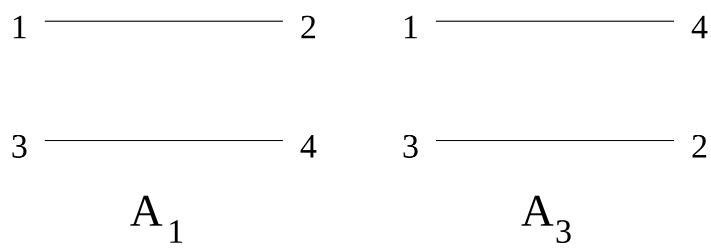

Figure 1: The disconnected graphs A_1 and A_3

and from the exchange graph B_2 of the scalar field α with dimension β

$$\beta = 2 - 2\eta(\varphi) - 2\kappa \tag{18}$$

Here κ, the "conformal dimension of the coupling constant" is of order $O(1/N)$ and can be expanded in powers of 1/N (see A. N. Vasilev et al. [5]). At leading order B_2 yields

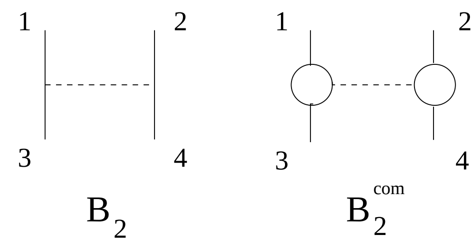

Figure 2: The graph B_2 and its complete radiatively corrected form B_2^{com}

$$(\vec{x}_{12}^2 \vec{x}_{34}^2)^{-\delta} \{C_1 u^{\mu-2\delta} F_\alpha(u,v) + C_2 F_{\vec{\varphi}^2}(u,v)\} \tag{19}$$

where the first (second) term describes the exchange of α ($\vec{\varphi}^2$), respectively, which are dual to each other in a representation theoretic sense. Expressions in (19) are

$$F_\alpha(u,v) = \sum_{n,m=0}^{\infty} \frac{u^n(1-v)^m}{n!m!} \frac{(n!)^2((n+m)!)^2}{(2n+m+1)!(3-\mu)_n} \tag{20}$$

$$F_{\vec{\varphi}^2}(u,v) = \sum_{n,m=0}^{\infty} \frac{u^n(1-v)^m}{n!m!} \frac{((\mu-1)_n(\mu-1)_{n+m})^2}{(2\mu-2)_{2n+m}(\mu-1)_n} \tag{21}$$

$$u = \frac{\vec{x}_{13}^2 \vec{x}_{24}^2}{\vec{x}_{12}^2 \vec{x}_{34}^2} \tag{22}$$

$$v = \frac{\vec{x}_{14}^2 \vec{x}_{23}^2}{\vec{x}_{12}^2 \vec{x}_{34}^2} \tag{23}$$

The constants $C_{1,2}$ involve the coupling constant z squared between α and $\vec{\varphi}$ to leading order z_1

$$z = \sum_{r=1}^{\infty} \frac{z_r}{N^r} \tag{24}$$

$$z_1 = 2\pi^{-2\mu} \frac{(\mu-2)\Gamma(2\mu-2)}{\Gamma(\mu)\Gamma(1-\mu)} \tag{25}$$

and are explicitly given as

$$C_1 = \frac{\Gamma(2\mu-1)}{\Gamma(\mu)^2\Gamma(1-\mu)\Gamma(\mu-1)(\mu-2)} \tag{26}$$

$$C_2 = -2 \tag{27}$$

In CFT in flat space conformal invariance determines three-point functions up to a few normalizing constants. The same must be true then for complete exchange graphs. For the radiatively corrected (at the vertices) graph B_2^{com} of B_2 we obtain instead of (19),(21)

$$C_2^{com}(\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}F_{\vec{\varphi}^2}^{com}(u,v) \tag{28}$$

with

$$C_2^{com} = -2 + O(1/N) \tag{29}$$

$$F_{\vec{\varphi}^2}^{com} = u^\kappa \sum_{n,m=0}^{\infty} \frac{u^n(1-v)^m}{n!m!} \frac{((\Delta)_n(\Delta)_{n+m})^2}{(2\Delta)_{2n+m}(2\Delta-\mu+1)_n} \tag{30}$$

$$\Delta = \delta + \kappa \tag{31}$$

where δ and κ are taken from (15) and (18). The part O(1/N) in C_2^{com} can stay undetermined (see below).

In the subsequent section we shall decompose the O(1) contribution to the Green function (12) into "conformal partial waves", amplitudes for the exchange of irreducible conformal fields. The O(1/N) contribution to this Green function contains three different types of terms: 1. Power series in u and $1-v$ containing the same conformal partial waves as the order O(1). These imply a renormalization of the coupling constants in the exchange amplitudes; 2. Power series in u and $1-v$ multiplied with $logu$. Consistency requires that conformal partial wave expansion of the power series yields the same exchange fields as the O(1) expansion. From the normalization we extract the combination

$$\frac{1}{2}(\eta(C)-\eta(A)-\eta(B)) \tag{32}$$

where the field C is exchanged and the fields A, B, C form a vertex of the exchange graph. 3. Power series in u and $1-v$ containing new conformal partial waves.

In the case of the Green function (12) the order O(1) gives the exchanged fields $J^{(l)}, l \geq 2$ even, and α. At O(1/N) there appear as new fields $J^{(l,1)}$, which contain the Goldstone fields. Since we are interested here only in the anomalous dimensions of $J^{(l)}$ it suffices to extract the $logu$ power series. These appear from the difference $B_2^{com} - B_2$ by expansion of the factor u^κ and from four new graphs B_1, B_3, C_{21}, C_{22}, where the first two $B_{1,3}$ are obtained by crossing B_2, C_{21} is a box graph, and the last one, C_{22}, is obtained by crossing C_{21}

The exchange graphs $B_1 + B_3$ are easily calculated and give ([6], equ. (3.17), only the $logu$ terms)

$$N^{-1}\frac{\mu}{\mu-2}\eta_1(\varphi)(\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}[-logu]\sum_{n,m=0}^{\infty}\frac{u^n(1-v)^m}{n!m!}$$
$$\left\{\frac{(\mu-1)_n)(\mu-1)_{n+m}(n+m)!}{(\mu)_{2n+m}} + \frac{((\mu-1)_{n+m})^2 n!}{(\mu)_{2n+m}}\right\} \tag{33}$$

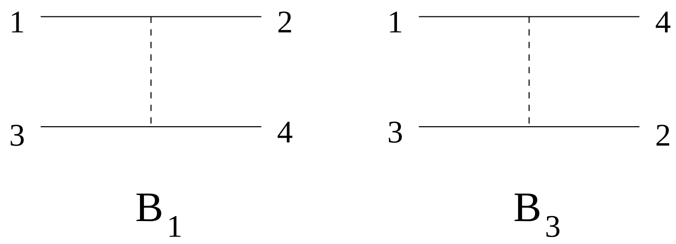

Figure 3: The crossed exchange graphs.

From the box graph C_{21} we get ([7], only the $logu$ terms)

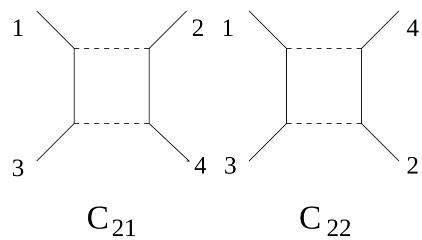

Figure 4: The box graphs.

$$N^{-1}\frac{\mu}{\mu-2}\eta_1(\varphi)2(2\mu-3)(\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}[-logu]\sum_{n,m=0}^{\infty}\frac{u^n(1-v)^m}{m!}$$
$$\frac{[(\mu-1)_{n+m}]^2}{(\mu)_{2n+m}}\sum_{p=0}^{n}\frac{(\mu-2)_p(\mu-1)_{n+m+p}}{p!(2\mu-3)_{n+m+p}} \tag{34}$$

To obtain the box graph C_{22} one can apply crossing explicitly by exchanging $\vec{x}_2$ with $\vec{x}_4$ or

$$u \longrightarrow \frac{u}{v} \qquad v \longrightarrow \frac{1}{v} \tag{35}$$
$$(\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}u^n(1-v)^m[-logu] \longrightarrow$$
$$(\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}u^n(1-v)^m[-logu+logv]\{(-1)^m\sum_{s=0}^{\infty}\frac{(n+m+\delta)_s}{s!}(1-v)^s\} \tag{36}$$

Inserting this into (34) we obtain the crossed graph contribution.

3 Deriving the anomalous dimensions

The Green function (12) can be submitted to a conformal partial wave decomposition by expressing it as a sum over exchange graphs (only the direct term) of α and $J^{(l)}$ at O(1), composites of two α and $J^{(l,1)}$ at O(1/N) and so on [8]. For example we consider exchange graphs of $J^{(l,t)}$ with conformal dimension $\Delta(l,t)$

$$(\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}u^{t+\frac{1}{2}(\eta(l,t)-2\eta(\varphi))}\sum_{n,m=0}^{\infty}\frac{u^n(1-v)^m}{n!m!}\alpha_{n,m}^{(l,t)} \tag{37}$$

Here we use an adhoc normalization

$$\alpha_{0,l}^{(l,t)} = 1 \tag{38}$$

If the external legs of the four-point function are all equal, the coefficients $\alpha_{n,m}^{(l,t)}$ are independent of the external fields (their dimension). Assume their dimension is δ. For the currents $J(l)$ these coefficients are particularly simple (since they belong to exceptional representations)

$$\alpha_{n,m}^{(l)} = \sum_{s=0}^{n}(-1)^s \binom{n}{s}\binom{m+n+s}{l}\frac{(\delta+l)_{m-l+n}(\delta+l)_{m-l+n+s}}{(2\delta+2l)_{m-l+n+s}} \tag{39}$$

These matrices for $n=0$ are easily inverted

$$\sum_{m=0}^{(l)} \beta_{l,m}\alpha_{0,m}^{(l')} = \delta_{l,l'} \tag{40}$$

with

$$\beta_{l,m} = (-1)^{l-m}\binom{l}{m}\frac{((\delta+m)_{l-m})^2}{(2\delta+m+l-1)_{l-m}} \tag{41}$$

This inversion formula was given in [8],(4.11), (4.12) in a more general version with two free parameters instead of only one (δ).

At order $O(1)$ we have the graphs $A_1 + A_3 + B_2$. We skip the F_α term and get for the remainder

$$(\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}[1+v^{-\delta}+C_2F_{\vec{\varphi}^2}] = (\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}\sum_{m,n=0}^{\infty}\frac{u^n(1-v)^m}{n!m!}a_{n,m} \tag{42}$$

where

$$a_{n,m} = \delta_{n,0}(\delta_{m,0}+(\delta)_m) - 2\frac{(\delta)_n((\delta)_{n+m})^2}{(2\delta)_{2n+m}} \tag{43}$$

We recognize immediately that

$$a_{0,0} = a_{0,1} = 0 \tag{44}$$

which implies that a current $J^{(l=0)}$ which would be identical with $\vec{\varphi}^2$ is not exchanged. The ansatz

$$a_{n,m} = \sum_{l=2,even}^{\infty} \gamma_l^2\alpha_{n,m}^{(l)} \tag{45}$$

is easily solved first by setting $n=0$ using the inversion formula (40) and then showing that it remains true for all n. The result is

$$\begin{aligned}\gamma_l^2 &= \frac{2((\delta)_l)^2}{(2\delta-1+l)_l} \qquad \text{for } l \geq 2, \text{ even}\\ &= 0 \qquad \text{for all other } l\end{aligned} \tag{46}$$

At order $O(1/N)$ we obtain the relevant $logu$ terms from

$$B_1 + B_3 + C_{21} + C_{22} + \{C_2^{com}F_{\vec{\varphi}^2}^{com} - C_2F_{\vec{\varphi}^2}\} \tag{47}$$

They sum up to

$$N^{-1}(\vec{x}_{12}^2\vec{x}_{34}^2)^{-\delta}\sum_{n,m=0}^{\infty}\frac{u^n(1-v)^m}{n!m!}[-b_{n,m}logu+c_{n,m}] \tag{48}$$

where $c_{n,m}$ is not known explicitly due to some nonevaluated integrals. But the $b_{0,m}$ (it suffices to give these only for $n=0$) are

$$\begin{aligned} b_{0,m} &= \frac{\mu(\mu-1)}{(\mu-2)[\mu-1+m]}\eta_1(\varphi) \\ &\{m!+(\mu-1)_m+2(2\mu-3)[\frac{((\mu-1)_m)^2}{(2\mu-3)_m} \\ &+m!\sum_{p=0}^{m}\frac{(\mu-1)_p(\mu-2)_p}{p!(2\mu-3)_p}]-2(4\mu-5)\frac{(\mu)_m(\mu-1)_m}{(2\mu-2)_m}\} \end{aligned} \tag{49}$$

Here we inserted κ_1, the coefficient of 1/N in the 1/N-expansion of κ, in the last term . But requiring consistence we must have

$$b_{0,0}=b_{0,1}=0 \tag{50}$$

which implies the known result

$$\kappa_1=\frac{\mu}{\mu-2}\eta_1(\varphi)(4\mu-5) \tag{51}$$

We recognize that the limit $d\to 3$ is rather delicate in (49) and that the point $d=3$ can only be reached by analytic continuation in d.

We solve next

$$b_{n,m}=\sum_{l\geq 2,even}[\eta_1(\varphi)-\frac{1}{2}\eta_1(l)]\gamma_l^2\alpha_{n,m}^{(l)} \tag{52}$$

only for $n=0$, and find after some algebra with the inverting matrix β (40), (41)

$$\eta_1(2) = 0 \tag{53}$$

$$\begin{aligned} \eta_1(l) &= \eta_1(\varphi)\frac{2(\mu-1)(2\mu+l-1)}{(2\mu-1)(\mu+l-2)_2}\{2(l-1) \\ &+ \sum_{p=1}^{\frac{1}{2}l-2}((p+1)!)^2\binom{l}{p+1}\frac{(2\mu+1+p)_{l-4-2p}}{(2\mu+1)_{l-4}}\} \qquad l\geq 4 \end{aligned} \tag{54}$$

Consistency requires that (52) is also solved for $n\neq 0$ by (53), (54). Of course $J^{(2)}$ is the energy-momentum tensor which should not have an anomalous dimension.

Now we set the dimension $d=3$. From

$$\eta_1(\varphi) = \frac{2\sin\pi\mu}{\pi}\frac{\Gamma(2\mu-2)}{\Gamma(\mu+1)\Gamma(\mu-2)} \tag{55}$$

$$\eta_1(\varphi)|_{d=3} = \frac{4}{3\pi^2} \tag{56}$$

we obtain

$$\eta_1(l)=\frac{16(l-2)}{3\pi^2(2l-1)} \tag{57}$$

Unitarity requires this anomalous dimension to be positive which it is indeed. Inserted into (7) we obtain the final mass formula

$$m(l)^2=\frac{16}{3N\pi^2}(l-2)+O(1/N^2) \tag{58}$$

4 Conclusion

The result (58) for the mass squared of the gauge bosons at leading order of 1/N is surprisingly simple due to cancellations between the five graphs primarily. Contrary to the case of the four-point function of the scalar field α, the four-point function (12) does apparently not reduce to an elementary function at $d = 3$ (see (34)). The linear dependence of the mass squared on the spin l of the boson is of Regge trajectory type and suggests the existence of a string theory from which the higher spin field theory on AdS(4) can be derived.

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